Possible C1 questions from past papers P1—P3

Source of the original question is given in brackets, e.g. [P1 January 2001 Question 1]; a question which has been edited is indicated with an asterisk, e.g. [P1 January 2001 Question 8*].

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1. Given that $(2 + \sqrt{7})(4 - \sqrt{7}) = a + b\sqrt{7}$, where a and b are integers,

(a) find the value of a and the value of b.

(2)

Given that $\dfrac{2+\sqrt{7}}{4+\sqrt{7}} = c + d\sqrt{7}$ where c and d are rational numbers,

(b) find the value of c and the value of d.

(3)

[P1 January 2001 Question 1]

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2. (a) Prove, by completing the square, that the roots of the equation $x^2 + 2kx + c = 0$, where k and c are constants, are $-k \pm \sqrt{(k^2 - c)}$.

(4)

The equation $x^2 + 2kx \pm 81 = 0$ has equal roots.

(b) Find the possible values of k.

(2)

[P1 January 2001 Question 2]

3.

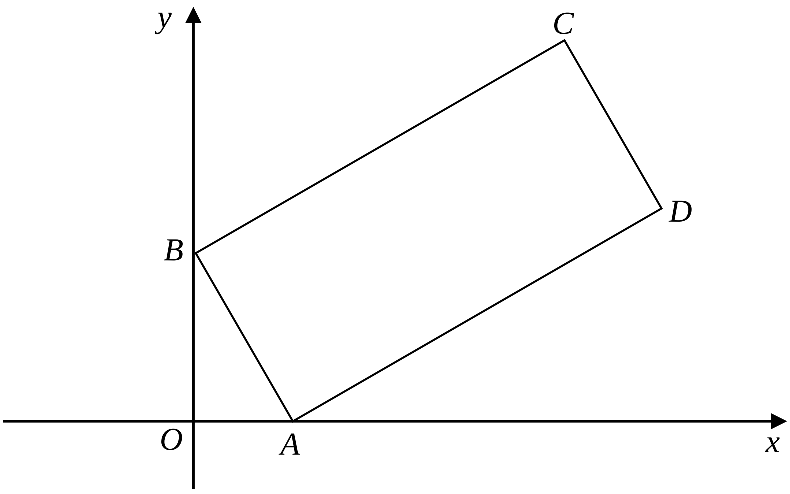

Fig. 2

The points *A* (3, 0) and *B* (0, 4) are two vertices of the rectangle *ABCD*, as shown in Fig. 2.

(*a*) Write down the gradient of *AB* and hence the gradient of *BC*.

(3)

The point *C* has coordinates (8, *k*), where *k* is a positive constant.

(*b*) Find the length of *BC* in terms of *k*.

(2)

Given that the length of *BC* is 10 and using your answer to part (*b*),

(*c*) find the value of *k*,

(4)

(*d*) find the coordinates of *D*.

(2)

[P1 January 2001 Question 6]

4.

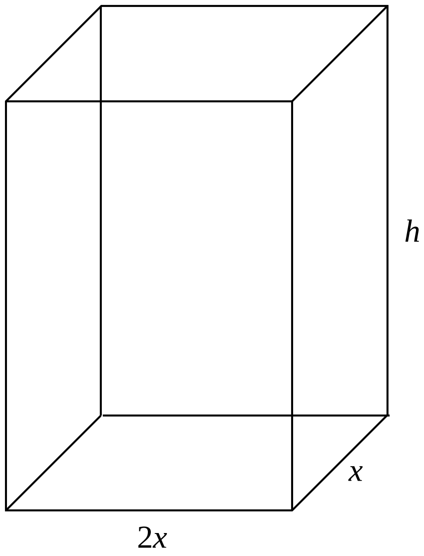

Fig. 4

A manufacturer produces cartons for fruit juice. Each carton is in the shape of a closed cuboid with base dimensions $2x$ cm by x cm and height h cm, as shown in Fig. 4.

Given that the capacity of a carton has to be 1030 cm^3,

(a) express h in terms of x,

(2)

(b) show that the surface area, A cm^2, of a carton is given by

$$A = 4x^2 + \frac{3090}{x}.$$

(3)

[P1 January 2001 Question 8*]

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5. (a) Given that $8 = 2^k$, write down the value of k. **(1)**

(b) Given that $4^x = 8^{2-x}$, find the value of x. **(4)**

[P1 June 2001 Question 1]

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6. The equation $x^2 + 5kx + 2k = 0$, where k is a constant, has real roots.

(*a*) Prove that $k(25k - 8) \geq 0$.

(2)

(*b*) Hence find the set of possible values of k.

(4)

(*c*) Write down the values of k for which the equation $x^2 + 5kx + 2k = 0$ has equal roots.

(1)

[P1 June 2001 Question 3]

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7. Each year, for 40 years, Anne will pay money into a savings scheme. In the first year she pays £500. Her payments then increase by £50 each year, so that she pays £550 in the second year, £600 in the third year, and so on.

(*a*) Find the amount that Anne will pay in the 40th year.

(2)

(*b*) Find the total amount that Anne will pay in over the 40 years.

(2)

Over the same 40 years, Brian will also pay money into the savings scheme. In the first year he pays in £890 and his payments then increase by £d each year.

Given that Brian and Anne will pay in exactly the same amount over the 40 years,

(*c*) find the value of d.

(4)

[P1 June 2001 Question 4]

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8. The points $A(-1, -2)$, $B\ (7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\Delta\ ABC$. Angle ABC is a right angle.

(a) Find the gradient of AB.

(2)

(b) Calculate the value of k.

(2)

(c) Show that the length of AB may be written in the form $p\sqrt{5}$, where p is an integer to be found.

(3)

(d) Find the exact value of the area of $\triangle\ ABC$.

(3)

(e) Find an equation for the straight line l passing through B and C. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

[P1 June 2001 Question 8*]

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9. Given that $2^x = \dfrac{1}{\sqrt{2}}$ and $2^y = 4\sqrt{2}$,

(a) find the exact value of x and the exact value of y,

(3)

(b) calculate the exact value of 2^{y-x}.

(2)

[P1 January 2002 Question 1]

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10. The straight line l_1 has equation $4y + x = 0$.

The straight line l_2 has equation $y = 2x - 3$.

(a) On the same axes, sketch the graphs of l_1 and l_2 . Show clearly the coordinates of all points at which the graphs meet the coordinate axes.

(3)

The lines l_1 and l_2 intersect at the point A.

(b) Calculate, as exact fractions, the coordinates of A.

(3)

(c) Find an equation of the line through A which is perpendicular to l_1. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

[P1 January 2002 Question 4]

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11. A curve C has equation $y = x^3 - 5x^2 + 5x + 2$.

(a) Find $\frac{dy}{dx}$ in terms of x.

(2)

The points P and Q lie on C. The gradient of C at both P and Q is 2. The x-coordinate of P is 3.

(b) Find the x-coordinate of Q.

(2)

(c) Find an equation for the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

This tangent intersects the coordinate axes at the points R and S.

(d) Find the length of RS, giving your answer as a surd.

(4)

[P1 January 2002 Question 5]

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12. Initially the number of fish in a lake is 500 000. The population is then modelled by the recurrence relation

$$u_{n+1} = 1.05u_n - d, \quad u_0 = 500\,000.$$

In this relation u_n is the number of fish in the lake after n years and d is the number of fish which are caught each year.

Given that $d = 15\,000$,

(a) calculate u_1 , u_2 and u_3 and comment briefly on your results.

(3)

Given that $d = 100\,000$,

(b) show that the population of fish dies out during the sixth year.

(3)

(c) Find the value of d which would leave the population each year unchanged.

(2)

[P2 January 2002 Question 5]

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13. (*a*) Find the sum of all the integers between 1 and 1000 which are divisible by 7.

(3)

(*b*) Hence, or otherwise, evaluate $\sum_{r=1}^{142}(7r+2)$.

(3)

[P1 June 2002 Question 1]

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14. Given that $f(x) = 15 - 7x - 2x^2$,

(*a*) find the coordinates of all points at which the graph of $y = f(x)$ crosses the coordinate axes.

(3)

(*b*) Sketch the graph of $y = f(x)$.

(2)

[P1 June 2002 Question 3*]

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15. (*a*) By completing the square, find in terms of k the roots of the equation

$$x^2 + 2kx - 7 = 0.$$ **(4)**

(*b*) Prove that, for all values of k, the roots of $x^2 + 2kx - 7 = 0$ are real and different.

(2)

(*c*) Given that $k = \sqrt{2}$, find the exact roots of the equation.

(2)

[P1 June 2002 Question 4]

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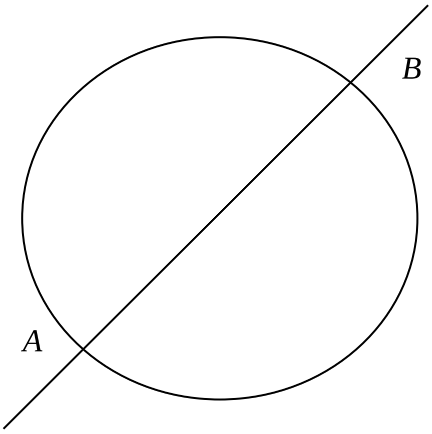
16. **Figure 3**



The points $A(-3, -2)$ and $B(8 , 4)$ are at the ends of a diameter of the circle shown in Fig. 3.

(*a*) Find the coordinates of the centre of the circle.
(2)

(*b*) Find an equation of the diameter AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

(*c*) Find an equation of tangent to the circle at B.
(3)

The line l passes through A and the origin.

(*d*) Find the coordinates of the point at which l intersects the tangent to the circle at B, giving your answer as exact fractions.
(4)

[P1 June 2002 Question 8]

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17. (*a*) Solve the inequality

$$3x - 8 > x + 13.$$
(2)

(*b*) Solve the inequality

$$x^2 - 5x - 14 > 0.$$
(3)

[P1 November 2002 Question 1]

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18. (a) An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is

$$\tfrac{1}{2}n[2a + (n - 1)d].$$

(4)

A company made a profit of £54 000 in the year 2001. A model for future performance assumes that yearly profits will increase in an arithmetic sequence with common difference £d. This model predicts total profits of £619 200 for the 9 years 2001 to 2009 inclusive.

(b) Find the value of d.

(4)

Using your value of d,

(c) find the predicted profit for the year 2011.

(2)

[P1 November 2002 Question 4*]

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19. $$f(x) = 9 - (x - 2)^2$$

(a) Write down the maximum value of f(x).

(1)

(b) Sketch the graph of y = f(x), showing the coordinates of the points at which the graph meets the coordinate axes.

(5)

The points A and B on the graph of y = f(x) have coordinates (–2, –7) and (3, 8) respectively.

(c) Find, in the form $y = mx + c$, an equation of the straight line through A and B.

(4)

(d) Find the coordinates of the point at which the line AB crosses the x-axis.

(2)

The mid-point of AB lies on the line with equation $y = kx$, where k is a constant.

(e) Find the value of k.

(2)

[P1 November 2002 Question 6]

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20. The curve C has equation $y = \mathrm{f}(x)$. Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 29$$

and that C passes through the point $P(2, 6)$,

(*a*) find y in terms of x.

(4)

(*b*) Verify that C passes through the point (4, 0).

(2)

(*c*) Find an equation of the tangent to C at P.

(3)

The tangent to C at the point Q is parallel to the tangent at P.

(*d*) Calculate the exact x-coordinate of Q.

(5)

[P1November 2002 Question 7]

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21. $$y = 7 + 10x^{\frac{3}{2}}.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(*b*) Find $\int y \;\mathrm{d}x$.

(3)

[P1 January 2003 Question 1]

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22. (*a*) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(2)

(*b*) Solve the simultaneous equations

$$x = 2y - 2,$$

$$x^2 = y^2 + 7.$$

(6)

[P1 January 2003 Question 3]

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23. The straight line l_1 with equation $y = \frac{3}{2}x - 2$ crosses the y-axis at the point P. The point Q has coordinates $(5, -3)$.

The straight line l_2 is perpendicular to l_1 and passes through Q.

(*a*) Calculate the coordinates of the mid-point of PQ.

(3)

(*b*) Find an equation for l_2 in the form $ax + by = c$, where a, b and c are integer constants.

(4)

The lines l_1 and l_2 intersect at the point R.

(*c*) Calculate the exact coordinates of R.

(4)

[P1 January 2003 Question 6]

24.

$$\frac{dy}{dx} = 5 + \frac{1}{x^2}.$$

(*a*) Use integration to find y in terms of x.

(3)

(*b*) Given that $y = 7$ when $x = 1$, find the value of y at $x = 2$.

(4)

[P1 June 2003 Question 1]

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25. Find the set of values for x for which

(*a*) $6x - 7 < 2x + 3$,

(2)

(*b*) $2x^2 - 11x + 5 < 0$,

(4)

(*c*) both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$.

(1)

[P1 June 2003 Question 2]

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26. In the first month after opening, a mobile phone shop sold 280 phones. A model for future trading assumes that sales will increase by x phones per month for the next 35 months, so that $(280 + x)$ phones will be sold in the second month, $(280 + 2x)$ in the third month, and so on.

Using this model with $x = 5$, calculate

(a) (i) the number of phones sold in the 36th month,

(2)

(ii) the total number of phones sold over the 36 months.

(2)

The shop sets a sales target of 17 000 phones to be sold over the 36 months.

Using the same model,

(b) find the least value of x required to achieve this target.

(4)

[P1 June 2003 Question 3]

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27. The points A and B have coordinates (4, 6) and (12, 2) respectively.

The straight line l_1 passes through A and B.

(*a*) Find an equation for l_1 in the form $ax + by = c$, where a, b and c are integers.

(4)

The straight line l_2 passes through the origin and has gradient –4.

(*b*) Write down an equation for l_2.

(1)

The lines l_1 and l_2 intercept at the point C.

(*c*) Find the exact coordinates of the mid-point of AC.

(5)

[P1 June 2003 Question 6]

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28. For the curve C with equation $y = x^4 - 8x^2 + 3$,

(*a*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(2)

The point A, on the curve C, has x-coordinate 1.

(*b*) Find an equation for the normal to C at A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

[P1 June 2003 Question 8*]

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29. The sum of an arithmetic series is

$$\sum_{r=1}^{n} (80 - 3r) .$$

(*a*) Write down the first two terms of the series.

(2)

(*b*) Find the common difference of the series.

(1)

Given that $n = 50$,

(*c*) find the sum of the series.

(3)

[P1 November 2003 Question 1]

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30. (*a*) Solve the equation $4x^2 + 12x = 0$.

(3)

$$\mathrm{f}(x) = 4x^2 + 12x + c,$$

where c is a constant.

(*b*) Given that $\mathrm{f}(x) = 0$ has equal roots, find the value of c and hence solve $\mathrm{f}(x) = 0$.

(4)

[P1 November 2003 Question 2]

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31. Solve the simultaneous equations

$$x - 3y + 1 = 0,$$

$$x^2 - 3xy + y^2 = 11.$$

(7)

[P1 November 2003 Question 3]

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32. A container made from thin metal is in the shape of a right circular cylinder with height h cm and base radius r cm. The container has no lid. When full of water, the container holds 500 cm^3 of water.

Show that the exterior surface area, A cm^2, of the container is given by

$$A = \pi r^2 + \frac{1000}{r}.$$

(4)

[P1 November 2003 Question 6*]

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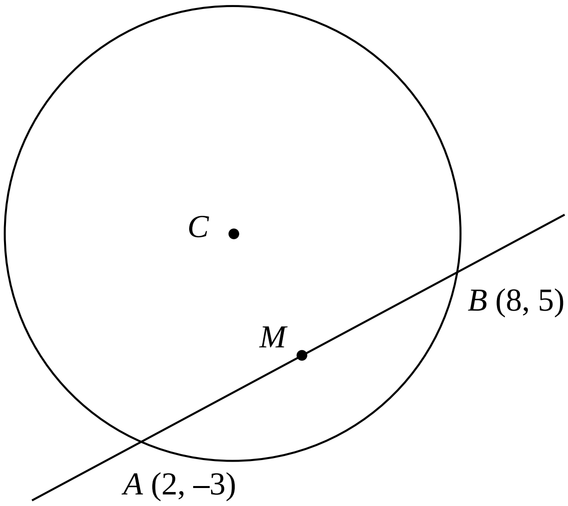
33. **Figure 1**



The points A and B have coordinates (2, –3) and (8, 5) respectively, and AB is a chord of a circle with centre C, as shown in Fig. 1.

(*a*) Find the gradient of AB.

(2)

The point M is the mid-point of AB.

(*b*) Find an equation for the line through C and M.

(5)

Given that the x-coordinate of C is 4,

(*c*) find the y-coordinate of C,

(2)

(*d*) show that the radius of the circle is $\frac{5\sqrt{17}}{4}$.

(4)

[P1 November 2003 Question 7]

34. The first three terms of an arithmetic series are p, $5p - 8$, and $3p + 8$ respectively.

(a) Show that $p = 4$.

(2)

(b) Find the value of the 40th term of this series.

(3)

(c) Prove that the sum of the first n terms of the series is a perfect square.

(3)

[P1 January 2004 Question 3]

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35. $$f(x) = x^2 - kx + 9, \text{ where } k \text{ is a constant.}$$

(a) Find the set of values of k for which the equation $f(x) = 0$ has no real solutions.

(4)

Given that $k = 4$,

(b) express $f(x)$ in the form $(x - p)^2 + q$, where p and q are constants to be found,

(3)

[P1 January 2004 Question 4*]

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36. The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3\sqrt{x} + \frac{12}{\sqrt{x}}, \quad x > 0.$$

(a) Show that, when $x = 8$, the exact value of $\frac{dy}{dx}$ is $9\sqrt{2}$.

(3)

The curve C passes through the point $(4, 30)$.

(b) Using integration, find $f(x)$.

(6)

[P1 January 2004 Question 5]

37.

Figure 2

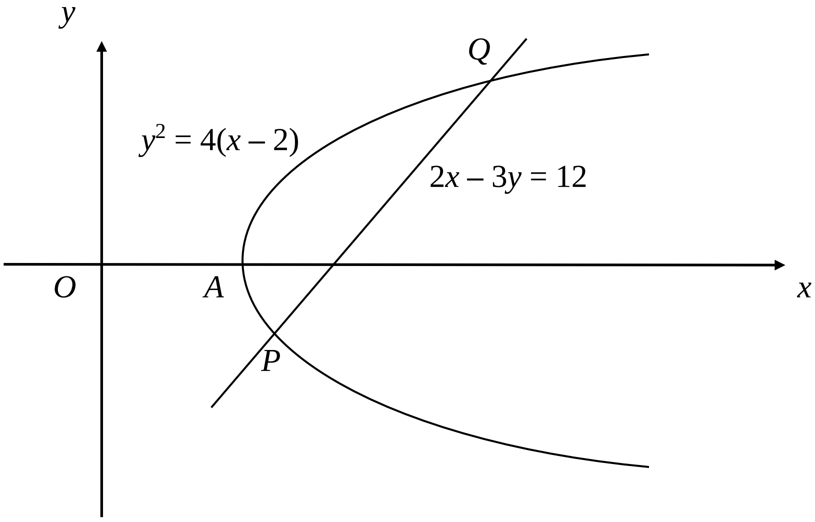

Figure 2 shows the curve with equation $y^2 = 4(x - 2)$ and the line with equation $2x - 3y = 12$.

The curve crosses the x-axis at the point A, and the line intersects the curve at the points P and Q.

(*a*) Write down the coordinates of A.

(1)

(*b*) Find, using algebra, the coordinates of P and Q.

(6)

(*c*) Show that $\angle PAQ$ is a right angle.

(4)

[P1 January 2004 Question 6]

38. A sequence is defined by the recurrence relation

$$u_{n+1} = \sqrt{\left(\frac{u_n}{2} + \frac{a}{u_n}\right)}, \quad n = 1, 2, 3, ...,$$

where a is a constant.

(a) Given that $a = 20$ and $u_1 = 3$, find the values of u_2, u_3 and u_4, giving your answers to 2 decimal places.

(3)

(b) Given instead that $u_1 = u_2 = 3$,

(i) calculate the value of a,

(3)

(ii) write down the value of u_5.

(1)

[P2 January 2004 Question 2]

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39. The points A and B have coordinates (1, 2) and (5, 8) respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

(b) Find, in the form $y = mx + c$, an equation for the straight line through A and B.

(4)

[P1 June 2004 Question 1]

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40. Giving your answers in the form $a + b\sqrt{2}$, where a and b are rational numbers, find

(a) $(3 - \sqrt{8})^2$,

(3)

(b) $\dfrac{1}{4 - \sqrt{8}}$.

(3)

[P1 June 2004 Question 2]

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41. The width of a rectangular sports pitch is x metres, $x > 0$. The length of the pitch is 20 m more than its width. Given that the perimeter of the pitch must be less than 300 m,

(a) form a linear inequality in x.

(2)

Given that the area of the pitch must be greater than 4800 m^2,

(b) form a quadratic inequality in x.

(2)

(c) by solving your inequalities, find the set of possible values of x.

(4)

[P1 June 2004 Question 3]

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42. The curve C has equation $y = x^2 - 4$ and the straight line l has equation $y + 3x = 0$.

(a) In the space below, sketch C and l on the same axes.

(3)

(b) Write down the coordinates of the points at which C meets the coordinate axes.

(2)

(c) Using algebra, find the coordinates of the points at which l intersects C.

(4)

[P1 June 2004 Question 4]

43.

$$f(x) = \frac{(x^2 - 3)^2}{x^3}, \; x \neq 0.$$

(a) Show that $f(x) \equiv x - 6x^{-1} + 9x^{-3}$.

(2)

(b) Hence, or otherwise, differentiate $f(x)$ with respect to x.

(3)

[P1 June 2004 Question 6*]
